Math 311/312 Cheat Sheet

For your information:

- $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$
- $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$
- $\sum_{k=0}^{\infty} \binom{r+k-1}{r-1} x^k = (1-x)^{-r}$ for $|x| < 1$
- $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
- $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
- $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\binom{n}{m} + \binom{n}{m+1} = \binom{n+1}{m+1}$
- $\sum_{k=0}^{n} \binom{n}{k} = 2^n$
- For any integer $n > 0$ and real $|r| < 1$
  - $\sum_{i=0}^{\infty} r^i = \frac{1}{1-r}$
  - $\sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$
  - $\sum_{i=0}^{n} r^i = \frac{1-r^{n+1}}{1-r}$

Distribution	$p(y)$	$\mathrm{E}(Y)$	$\mathrm{Var}(Y)$	$\mathrm{M}(s)$
binomial	$p(y) = \binom{n}{y} p^y (1-p)^{n-y}$ $y = 0, 1, 2, \ldots, n$	np	$np(1-p)$	$[pe^s + (1-p)]^n$
geometric	$p(y) = p(1-p)^{y-1}$ $y = 1, 2, \ldots$	$\frac{1}{p}$	$\frac{(1-p)}{p^2}$	$\frac{pe^s}{1-(1-p)e^s}$
hypergeometric	$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$ $y = 0, 1, \ldots, n$ if $n \le r$, $y = 0, 1, \ldots, r$ if $n > r$	$\frac{nr}{N}$	$n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$	
Poisson	$p(y) = \frac{\lambda^y e^{-\lambda}}{y!}$ $y = 0, 1, 2, \ldots$	λ	λ	$\exp[\lambda(e^s - 1)]$
negative binomial	$p(y) = \binom{y-1}{r-1} p^r (1-p)^{y-r}$ $y = r, r+1, \ldots$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[\frac{pe^s}{1-(1-p)e^s}\right]^r$

Distribution	$f(y)$	E(Y)	Var(Y)	M(s)
uniform	$f(y) = \frac{1}{\theta_2 - \theta_1}$ $\theta_1 \leq y \leq \theta_2$	$\frac{\theta_1+\theta_2}{2}$	$\frac{(\theta_2-\theta_1)^2}{12}$	$\frac{e^{s\theta_2}-e^{s\theta_1}}{s(\theta_2-\theta_1)}$
normal	$f(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{1}{2\sigma^2}\right)(y-\mu)^2\right]$ $-\infty < y < \infty$	μ	σ^2	$\exp\left(\mu s + \frac{s^2\sigma^2}{2}\right)$
exponential	$f(y) = \frac{1}{\beta} e^{-y/\beta} \quad \beta > 0$ $0 < y < \infty$	β	β^2	$(1-\beta s)^{-1}$
gamma	$f(y) = \left[\frac{1}{\Gamma(\alpha)\beta^\alpha}\right] y^{\alpha-1} e^{-y/\beta}$ $0 < y < \infty$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta s)^{-\alpha}$
chi-square	$f(y) = \frac{y^{(\nu/2)-1} e^{-y/2}}{2^{\nu/2}\Gamma(\nu/2)}$ $y > 0$	ν	2ν	$(1-2s)^{-\nu/2}$
beta	$f(y) = \left[\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\right] y^{\alpha-1}(1-y)^{\beta-1}$ $0 < y < 1$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	

- For X_i iid $f_X(x)$ where f is an exponential family distribution, and an unbiased estimator $\widehat{\theta}$ of θ, the CRLB is $\text{Var}(\widehat{\theta}) \geq 1/I_n(\theta)$ where

$$\begin{aligned} I_n(\theta) &= nE\left(-\frac{d^2 \ln(f_X(x))}{d\theta^2}\right) \\ &= E\left(-\frac{d^2 \ln(f_{\mathbf{X}}(\mathbf{x}))}{d\theta^2}\right) \\ &= nE\left[\left(\frac{d \ln(f_X(x))}{d\theta}\right)^2\right] \\ &= E\left[\left(\frac{d \ln(f_{\mathbf{X}}(\mathbf{x}))}{d\theta}\right)^2\right] \end{aligned}$$

- The mean squared error of an estimator $\widehat{\theta}$ of θ is

$$\begin{aligned} MSE\left(\hat{\theta}\right) &= \text{E}\left[\left(\hat{\theta} - \theta\right)^2\right] \\ &= \text{Var}\left(\hat{\theta}\right) + \left(\text{bias}_{\hat{\theta}}(\theta)\right)^2 \end{aligned}$$